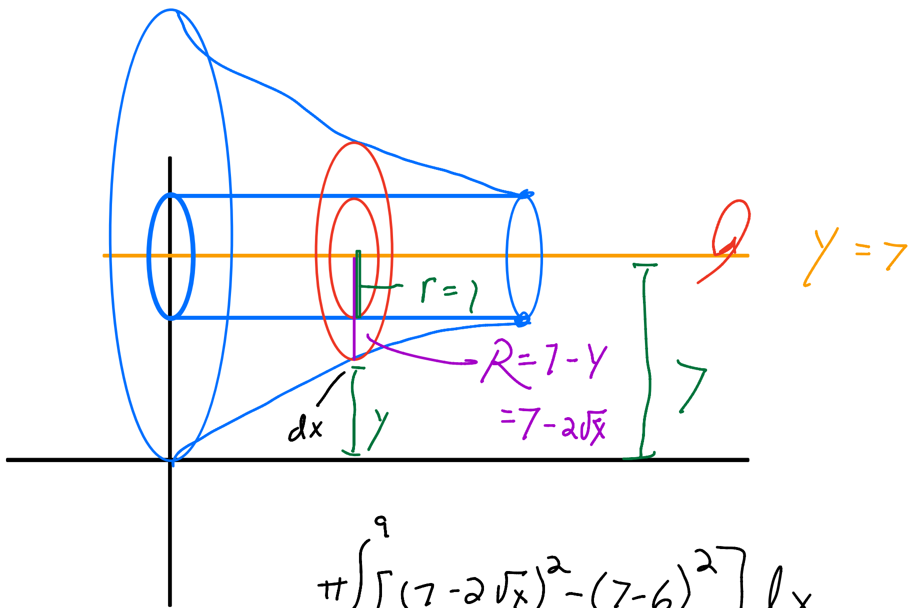
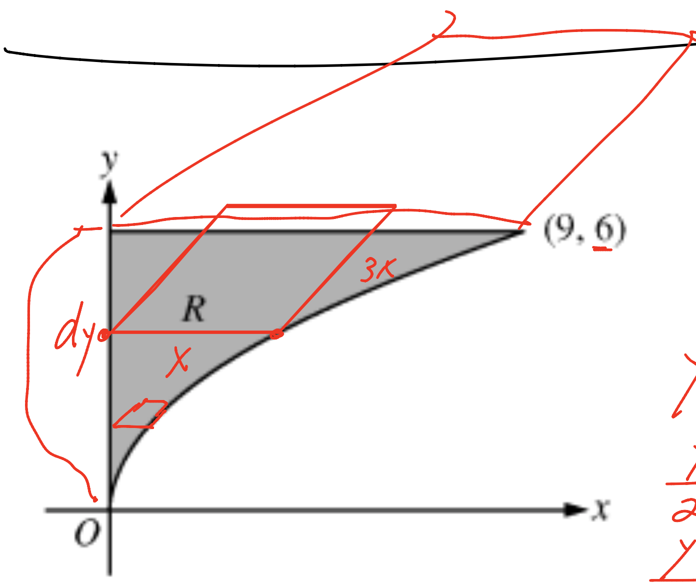
Period 1, April 21, 2025



$$\pi\int_0^9 \left[(7-2\sqrt{x})^2-(7-6)^2\right]dx$$



$$\int_0^6 x\cdot 3x\cdot dy = \int_0^6 \frac{y^2}{4}\cdot 3\cdot \frac{y^2}{4}\,dy$$

$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$\frac{y^2}{4} = x$

$$\int_0^6 \frac{3y^4}{16}\,dy = \text{Volume}$$

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region

$$\int \left(4\cos \tfrac{1}{4}\pi x - (2x^2 - 6x + 4) \right) dx$$

$$\int (4\cos \frac{\pi x}{4} - 2x^2 + 6x - 4)\, dx$$

$$4\cos \frac{\pi x}{4}$$

$$U = \frac{\pi x}{4}$$

$$dU = \frac{\pi}{4} \cdot dx$$

$$\frac{4\,dU}{\pi} = dx$$

$$\int 4\cos \frac{\pi x}{4}\, dx$$

$$\int 4\cos U \cdot \frac{4\,dU}{\pi} = \frac{16}{\pi} \int \cos U\, dU$$

$$\frac{16}{\pi}\sin U = \frac{16}{\pi}\sin \frac{\pi x}{4}$$

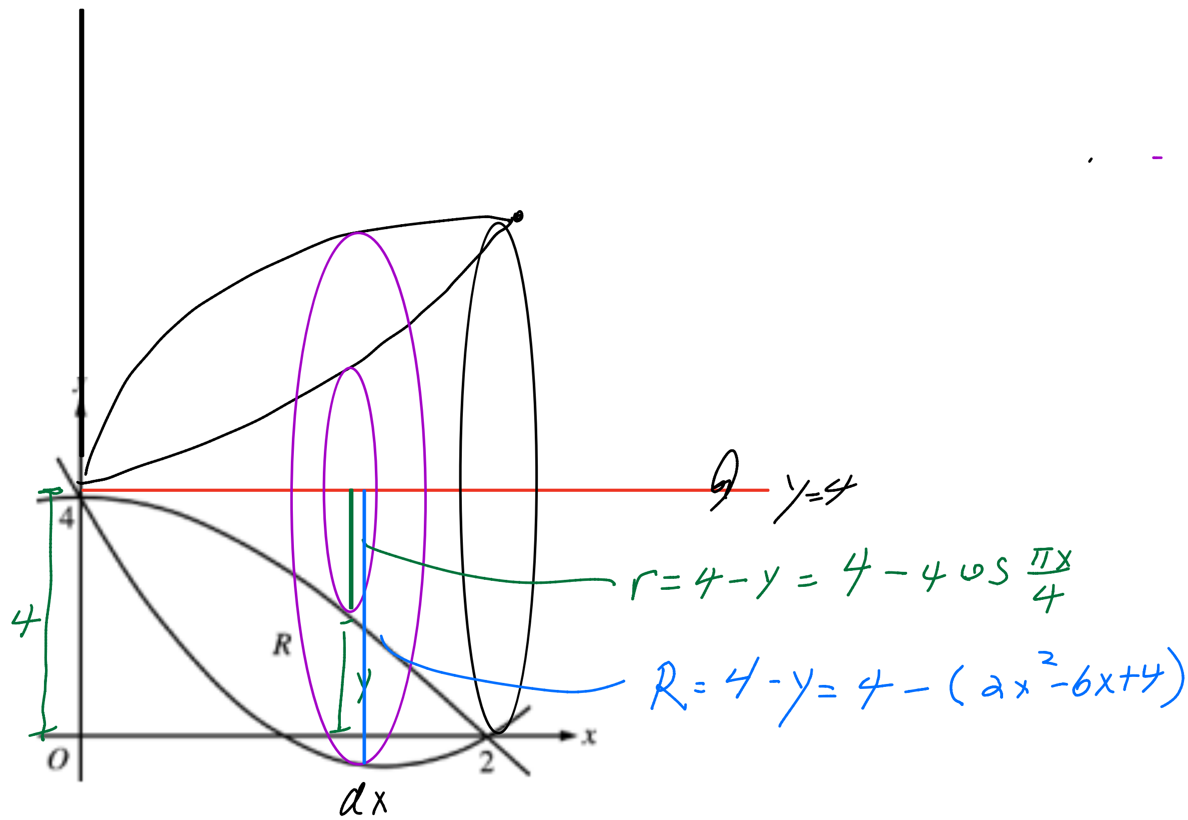
$$\int_0^2 (4\cos \frac{\pi x}{4} - 2x^2 + 6x - 4)\, dx = \frac{16}{\pi}\sin \frac{\pi x}{4} - \frac{2}{3}x^3 + 3x^2 - 4x + C \Big]_0^2$$

$$\frac{16}{\pi} \cdot \sin \frac{2\pi}{4} - \frac{2}{3}(2)^3 + 3(2)^2 - 4(2) - \left[\frac{16}{\pi}\sin \frac{0 \cdot \pi}{4} - \frac{2}{3}(0)^3 + 3(0)^2 - 4(0) \right]$$

$$0 \quad 0 \quad 0 \quad 0$$

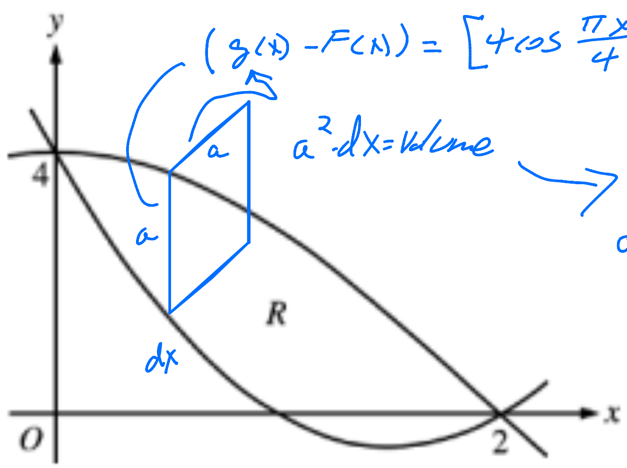
$$\frac{16}{\pi}\sin \frac{\pi}{2} - \frac{16}{3} + 12 - 8 - 0$$

$$\frac{16}{\pi} \cdot 1 - \frac{16}{3} + 4 = \frac{16}{\pi} - \frac{16}{3} + \frac{12}{3} = \underbrace{\frac{16}{4} - \frac{4}{3}}$$



$$\pi\int_0^2 \left(R^2 - r^2\right)dx = \pi\int_0^2 \left[\left(4-(2x^2-6x+4)\right)^2 - \left(4-4\cos\frac{\pi x}{4}\right)^2\right]dx$$

$$\pi\int_0^2 \left[\left(-2x^2+6x\right)^2 - \left(4-4\cos\frac{\pi x}{4}\right)^2\right]dx$$



$(g(x) - F(x)) = \left[4\cos\frac{\pi x}{4} - (2x^2 - 6x + 4)\right]$

$a^2 dx = \text{Volume}$

$\rightarrow \int_0^2 \left[4\cos\frac{\pi x}{4} - (2x^2-6x+4)\right]^2 dx$

2. (1985 BC3-appropriate for AB)

$$\int_1^2 \frac{x+1}{x^2+2x}\,dx =$$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

$u = x^2 + 2x$

$du = (2x+2)dx$

$\frac{du}{2x+2} = dx$

$\frac{du}{2(x+1)} = dx$

$\int \frac{\cancel{x+1}}{u} \cdot \frac{du}{2\cancel{(x+1)}} = \frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|x^2+2x| + C \Big[_1^2$

$\frac{1}{2}\ln|\underbrace{2^2+2\cdot 2}_{\substack{4+4 \\ 8}}| - \frac{1}{2}\ln[\underbrace{1^2+2(1)}_{\substack{1+2 \\ 3}}]$

$\frac{1}{2}\ln 8 - \frac{1}{2}\ln 3 = \frac{\ln 8 - \ln 3}{2}$

$$\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx =$$

$U = 1 - x^2$

$dU = -2x\,dx$

$\frac{dU}{-2x} = dx$

$\int \frac{\cancel{2x}}{\sqrt{U}} \cdot \frac{dU}{\cancel{-2x}} = -\int \frac{dU}{\sqrt{U}} = -\int U^{-\frac{1}{2}}\,dU$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

$-\left[\frac{2}{1} U^{-\frac{1}{2}+1=\frac{1}{2}} + C\right]$

$-2\sqrt{U} + C$

$-2\sqrt{1-x^2}\Big|_0^{\frac{1}{2}}$

$-2\sqrt{1-\left(\frac{1}{2}\right)^2} - \left(-2\sqrt{1-0^2}\right)$

$-2\sqrt{1-\frac{1}{4}} + 2\sqrt{1}$

$-2\sqrt{\frac{3}{4}} + 2$

$\frac{-\cancel{2}\sqrt{3}}{\cancel{2}} + 2 = 2 - \sqrt{3}$

If $\int_0^k \left(2kx - x^2\right)dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

$\cancel{2}k \cdot \frac{1}{\cancel{2}} \cdot x^{1+1=2} - \frac{1}{3}x^3 \Big|_0^k = 18$

$k \cdot x^2 - \frac{1}{3}x^3 \Big|_0^k = 18$

$k \cdot k^2 - \frac{1}{3} \cdot k^3 - \cancel{\left[k \cdot 0^2 - \frac{1}{3}(0)^3\right]} = 18$

$k^3 - \frac{1}{3}k^3 = \frac{2}{3}k^3 = 18$

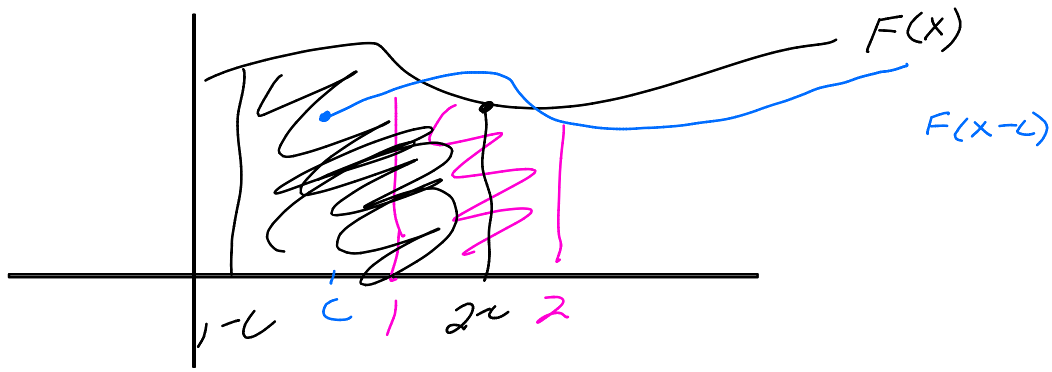
$\cancel{\frac{3}{2}} \cdot \cancel{\frac{2}{3}} k^3 = 18 \cdot \frac{3}{2}$

$k^3 = 27 = 3^3$

$k = 3$

If $\int_{1}^{2} f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5



If the substitution $u = \frac{x}{2}$ is made, the integral $\int_{2}^{4} \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

$u = \frac{x}{2} \Rightarrow 2u = x$

$du = \frac{1}{2}dx$

$2\,du = dx$

(A) $\int_{1}^{2} \frac{1-u^2}{u}\,du$ (B) $\int_{2}^{4} \frac{1-u^2}{u}\,du$ (C) $\int_{1}^{2} \frac{1-u^2}{2u}\,du$

(D) $\int_{1}^{2} \frac{1-u^2}{4u}\,du$ (E) $\int_{2}^{4} \frac{1-u^2}{2u}\,du$

$x=4$

$\frac{x}{2} = 2$

$\int_{1}^{2} \frac{1-u^2}{\cancel{2}u} \cdot \cancel{2}\,du$

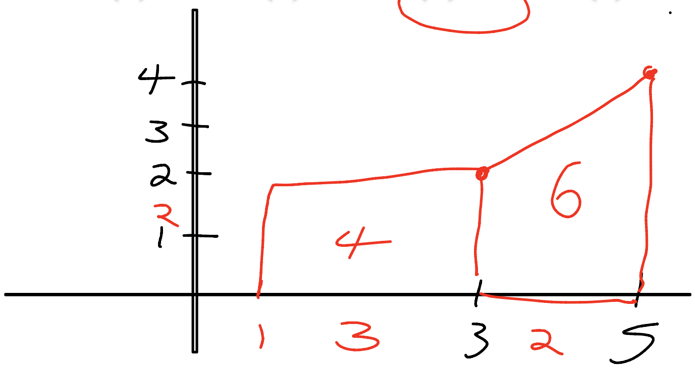
$x=2$

$\frac{x}{2} = 1$

9. (2012 AB13)

The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12



$\frac{1}{2}(2+4)2$

6. (1973 AB30)

$\int_1^2 \frac{x-4}{x^2}\,dx = \int_1^2 \left(\frac{x}{x^2} - \frac{4}{x^2}\right)dx = \int_1^2 \left(\frac{1}{x} - 4 \cdot x^{-2}\right)dx$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

$\ln|x| - 4 \cdot \frac{1}{-1} \cdot x^{-2+1=-1} = \ln|x| + 4 \cdot x^{-1}$

$= \ln|x| + \frac{4}{x} \Big|_1^2$

$\ln|2| + \frac{4}{2} - \left[\ln|1| + \frac{4}{1}\right]$

$\ln 2 + 2 - (0 + 4) = \ln 2 + 2 - 4 = \ln 2 - 2$

8. (2012 AB12)

Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^4 e^u\,du$ (C) $2\int_1^2 e^u\,du$ (D) $\frac{1}{2}\int_1^2 e^u\,du$ (E) $\int_1^4 e^u\,du$

$u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{\frac{1}{2}-1}dx$

$du = \frac{1}{2\sqrt{x}}dx$

$du \cdot 2\underbrace{\sqrt{x}}_{u} = dx$

$x = 4$

$\sqrt{x} = 2$

$x = 1$

$\sqrt{x} = 1$

$\int_1^2 \frac{e^u}{\cancel{u}} \cdot du \cdot 2\cancel{u} = 2\int_1^2 e^u du$

12. (1969 AB38)

$\int \frac{x^2}{e^{x^3}}\,dx =$

$u = x^3$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$\int \frac{\cancel{x^2}}{e^u} \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int e^{-u} du$

$L = -u$

$dL = -du$

$-dL = du$

$\frac{1}{3}\int e^L \cdot -dL$

$-\frac{1}{3}e^L = -\frac{1}{3}e^{-u} = -\frac{1}{3}e^{-x^3}$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

$-\frac{1}{3}e^{-x^3} + C = \frac{-1}{3e^{x^3}} + C$

$$\int_a^b (F(x)+5)\,dx = \int_a^b F(x)\,dx + \int_a^b 5\,dx$$

$a+2b$ $\quad$ $5x\Big|_a^b$

$5b-5a$

14. (1997 AB3)

If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

$a+2b+5b-5a$

$7b-4a$

11. (1969 AB29)

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx =$$

$u=\sin x$ $\quad$ $\int \frac{du}{\cos x}=dx$

$du=\cos x\,dx$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

$$\int \frac{\cancel{\cos x}}{u}\cdot\frac{du}{\cancel{\cos u}} = \int \frac{du}{u} = \ln|u|+C = \ln|\sin x| \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\ln\left|\sin\tfrac{\pi}{2}\right| - \ln\left|\sin\tfrac{\pi}{4}\right| = \ln 1 - \ln\tfrac{\sqrt{2}}{2} = 0 - \ln\tfrac{\sqrt{2}}{2}$$

$$-(\ln\sqrt{2}-\ln 2) = -(\ln 2^{\frac{1}{2}} - \ln 2)$$

$\frac{1}{2}x - x = -\frac{1}{2}x$

$$-\left(\tfrac{1}{2}\ln 2 - \ln 2\right)$$

$$-\left(\underline{-\tfrac{1}{2}\ln 2}\right) = +\tfrac{1}{2}\ln 2 = \ln\sqrt{2}$$

$\int \sin(2x+3)\,dx =$

$u = 2x+3$
$du = 2dx$
$\frac{du}{2} = dx$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

$\int \sin u \cdot \frac{du}{2} = \frac{1}{2}\int \sin u\,du = \frac{1}{2}(-\cos u)+C$

$-\frac{1}{2}\cos(2x+3)+C$

15. (1997 BC1-appropriate for AB)

$\int_0^1 \sqrt{x}\,(x+1)\,dx =$

$\int_0^1 (x^{\frac{3}{2}}+x^{\frac{1}{2}})dx = \frac{2}{5}x^{\frac{3}{2}+1=\frac{5}{2}} + \frac{2}{3}x^{\frac{1}{2}+1=\frac{3}{2}}$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

$\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2}\Big]_0^1$

$\frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2} - \left[\cancel{\frac{2}{5}(0)^{5/2} + \frac{2}{3}(0)^{3/2}}\right]$

$\frac{3\cdot 2}{3\cdot 5} + \frac{2\cdot 5}{3\cdot 5} = \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$

34, 35

$$\int_3^5 T\left(\sqrt{2T-1}\right) dT$$

$X = 2T - 1 \Rightarrow \frac{X+1}{2} = T$

$dx = 2dT$

$\frac{dx}{2} = dT$

$5 = T$

$10 - 1 = 2T - 1 = 9$

$T = 3$

$6 - 1 = 2T - 1 = 5$

$k, a, b = \frac{1}{4}, 5, 9$

B

$$\int_5^9 T \cdot \sqrt{x} \cdot \frac{dx}{2}$$

$$\int_5^9 \frac{x+1}{2} \cdot \sqrt{x} \cdot \frac{dx}{2}$$

$$\frac{1}{4} \int_5^9 (x+1)\sqrt{x}\, dX$$